The effect of some winding defects on the output of a Rogowski coil
Part 2 - Real coils

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Abstract

An ideal, perfectly-wound, Rogowski coil should respond only to currents which thread the coil. It should not matter if the current-carrying conductor is off axis or threads the coil obliquely. Furthermore, the coil should have zero response to electric currents and other sources of magnetic fields which are external to the loop of the coil. These properties follow as a consequence of Ampère's Law.

In a previous Note the basic concepts relating to a gap in the winding of an otherwise perfect coil were discussed for the special case where the plane of the coil is perpendicular to the conductor. This Note looks at some more practical aspects. Methods are described for minimising the effect of the gap in the winding of a flexible coil where the ends are joined. It is also shown how other winding defects such as bumps or hollows in the coil former can be evaluated. The concepts involved when a coil is not oriented perpendicular to the conductor are also discussed.

Keywords: Rogowski coil, mutual inductance, measurement, Ampère's law, electric current, pick-up, cross-talk, winding imperfection.

List of symbols

A	cross-sectional area of a coil former
χ	angle of skew (Fig 13)
D	distance of an external conductor from the gap in a coil (Fig 8)
d	overall diameter of a coil cross-section (including the insulation)
α	angle subtended at the conductor by a section of winding
g	winding gap
H	magnetic field
I	current being measured by the coil
L	length of a section of flexible coil
λ	angle of tilt between a coil and the plane perpendicular to the conductor (Fig 7)
M	mutual inductance of a coil
μ_0	permittivity of free space
n	turns density (turns/metre)
r	distance of a section of coil from the conductor (Fig 2)
S	'strength' of a section of coil (Eq 2)
S_d	'strength' of a defective section of winding (Sect 5.1)
θ	angular position of a section of coil (Fig 2)
V	voltage developed in a Rogowski coil
z	cylindrical coordinate axis (Fig 2)

1. Introduction

Rogowski coils are now widely used for electric current measurements in many applications ranging from power quality monitoring of buildings to the measurement of lightning strikes, very large currents in arc furnaces and switchgear testing. They have several advantages over conventional current transformers. A well-made coil with a near-perfect winding can come close to the ideal for a current transducer in that it responds only to currents that thread the loop of the coil and is not sensitive to the distribution of currents within the loop. Furthermore, it will not respond to electric currents or other sources of magnetic field which are outside the loop of the coil.

A 'perfect' winding means that the cross-section of the coil former is exactly constant round the coil and the turns density of the winding is also exactly uniform. Although considerable efforts are made in the construction of coils it is inevitable that some imperfections will always remain.

In a previous Note, Part 1 [Ref. 1] we provided a quantitative relationship between winding defects and the resulting measurement errors when the coil is used to measure the current in a conductor. This Note develops these ideas further to deal with some practical situations such as the winding gap where the ends of a flexible coil are joined, other winding defects such a bumps or hollows in the coil former material and when the plane of a coil is not perpendicular to the conductor.

2. Coil Construction

The Rogowski coil is an ‘air cored’ toroidal winding placed round the conductor in such a way that the alternating magnetic field produced by the current induces a voltage in the coil [Refs. 2, 3]. Despite its name this technique was first described by Chattock in 1887 [Ref. 4].

Rogowski coils are generally classed as either rigid or flexible. For a rigid coil the coil former is in the shape of a toroid made of a hard material such as plastic. With flexible coils the winding is placed on a flexible former, such as rubber, which is then bent round to form a toroid. This Note is concerned with coils having a winding with a small cross section which are best approximated by a flexible coil.

2.1 The Reverse Turn

As well as the large number of small turns that make up the winding, a Rogowski coil can also be seen as one large loop which can have a voltage induced in it by magnetic fields with a component perpendicular to the plane of the coil. This can result in large pick-up errors from external sources of magnetic field. The solution is to use a reverse turn which returns along the length of the coil. The voltage induced in the reverse turn cancels out the 'large loop' voltage.

A reverse turn has the additional advantage that the connections to the coil are at one end only. The coil has a 'free' end which can easily be threaded round the conductor being measured.

The use of a reverse turn is a very well-known technique. In the discussion which follows about winding defects it is assumed that the coil has a reverse turn.

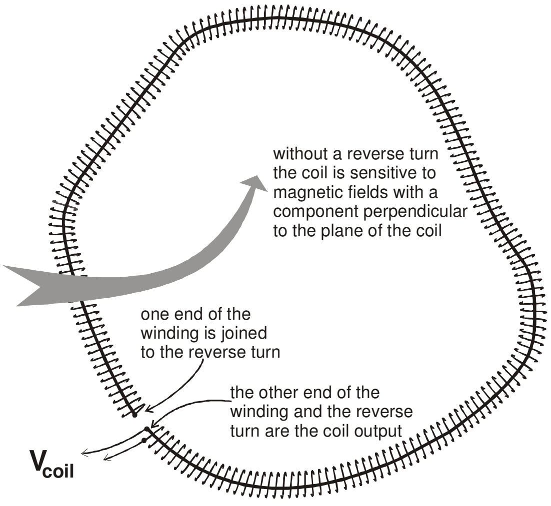


Figure 1: Arrangement for the reverse turn

2.2 Winding Defects

Winding defects include missing turns and gaps in the winding as well as non-uniformities in the cross-sectional area of the coil former. In Figure 1, for example, there is a gap between the ends of the winding. This gap will cause the coil to be sensitive to external magnetic fields which have a component in the plane of the coil. It will also make the output of the coil sensitive to the position of the conductor being measured within the coil.

3. Analysis

3.1 Basic Theory

The basic theory concerning gaps in windings was given in detail in Part 1 [Ref. 1]. This theory is essential to an understanding of the present work and a summary will be given here. Unless otherwise stated all the angles are expressed in radians.

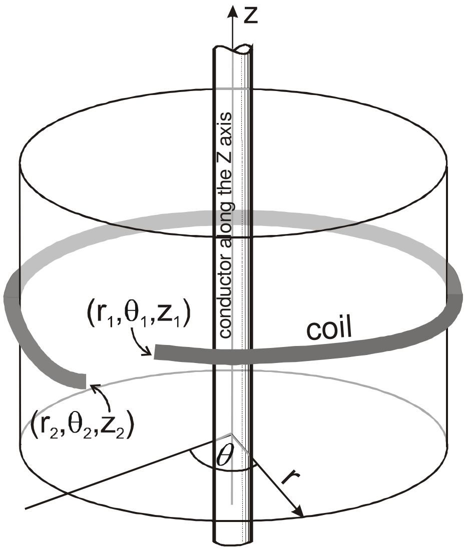


Figure 2: Coordinate system

The position of the coil is described in cylindrical co-ordinates (r, θ, z). The coordinates of the ends of the coil are (r_1, θ_1, z_1) and (r_2, θ_2, z_2) (Figure 2). With respect to coil position the mutual inductance, M, between the coil and the conductor depends only on θ_1 and θ_2 and is given by:

$$M = \frac{\mu_0 nA}{2\pi}(\theta_2 - \theta_1) = \frac{\mu_0 nA}{2\pi}\alpha = S\alpha \qquad (1)$$

where $\alpha = \theta_2 - \theta_1$. A is the cross-sectional area of the winding and n is the turns density. The concept of the 'Strength', S, of a winding is introduced where:

$$\text{Strength} = S = \frac{\mu_0 nA}{2\pi} \qquad (2)$$

The analysis makes no assumptions about the shape of the coil i.e whether it is circular or not, or whether the ends are in the same plane perpendicular to the Z axis.

This result applies provided the coil is not too close to the conductor. If the distance from the centre of the conductor is more than about 7.5 x the cross-sectional diameter of the coil former, the error is small (<0.1%). If the distance is about 2.5 times the diameter, the error is about 1%.

Equation 1 shows that the mutual inductance between a Rogowski coil winding and a conductor depends only on the 'strength' of the winding and the angle subtended at the conductor by the ends of the winding.

4. Gap Compensation

When a flexible coil is wrapped round a conductor there is inevitably a gap in the winding where the ends come together. This is a source of error in that the coil becomes sensitive to the position of the conductor within the coil and is also susceptible to interference from external conductors.

As an example we consider a circular coil of diameter 160mm with a gap of 10mm between the ends. If the conductor is moved from the centre of the coil towards the gap by a distance of 40mm (half the coil radius) there will be a change in output of about 2%. If it is moved from the centre by 60mm the change is about 6%.

To minimise these errors it is possible to incorporate some form of compensation into the coil design. Two methods are considered here: 1) the use of a compensation coil, and 2) overlapping ends.

4.1 Compensation Coil

The principle of a compensation coil is illustrated in Figure 3. An extra winding is placed over the coil winding near the end of the coil to match the winding that is 'missing' fom the gap. Referring to equation 1, with $\alpha = \theta_2 - \theta_1$, the compensation coil should follow the relationship:

$$n_c A_c \alpha_c = n_g A_g \alpha_g \ . \qquad (3)$$

α_c is the angle subtended by the compensation coil and α_g is the angle subtended by the gap. n_c and n_g are the turns densities of the compensation coil and the winding 'missing' from the gap. The cross-sectional area of the compensation coil, A_c, will be larger than the area of the winding missing from the gap, A_g, because it is wound over the top of the main coil winding.

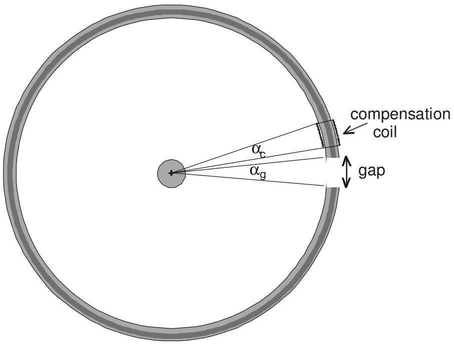


Figure 3: Compensation Coil

Using the methods described in Part 1 it is possible to calculate the effect of the compensating coil in reducing the errors due to off-centre operation.

Figure 4 shows a coil with a compensation winding designed to compensate for the gap when the conductor is at the centre of the coil. The calculation assumes a coil with a diameter of 160mm and a gap width of 10mm. The change in output is calculated for several different off-centre conductor positions and the results are presented in Table 1. It is seen that the compensation coil gives a substantial improvement.

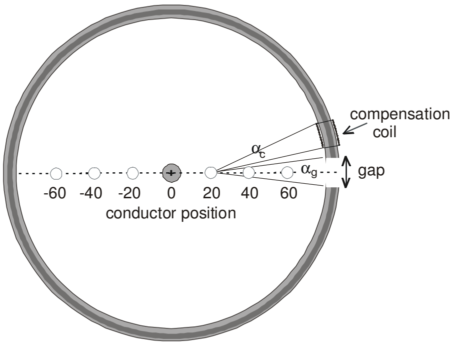


Figure 4: Internal conductor positions

Conductor distance from the centre (mm)	Change in output	
	without compensation	with compensation
-60	-0.85%	0.00%
-40	-0.66%	0.00%
-20	-0.40%	0.00%
0	0.00%	0.00%
20	0.66%	0.02%
40	1.98%	0.17%
60	5.87%	1.79%

Table 1: Change in output vs. conductor position

A similar calculation can be made to determine the effect of a compensation coil on the pick-up from an external conductor (Figure 5). Table 2 shows the results. The compensation coil gives a significant improvement in reducing external pick-up.

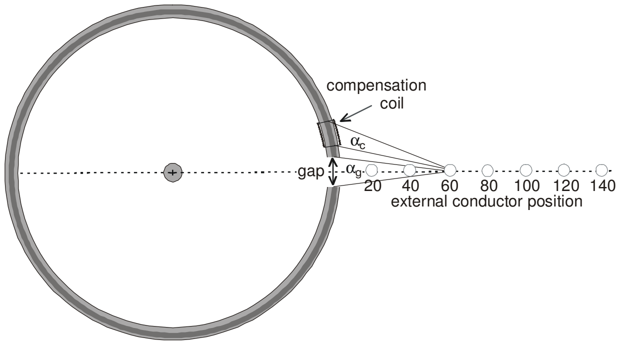


Figure 5: External conductor positions

External conductor position	Pick-up from the external conductor	
	without compensation	with compensation
20	7.86%	1.79%
40	3.97%	0.17%
60	2.65%	0.02%
80	1.99%	0.00%
100	1.59%	0.00%
120	1.33%	0.00%
140	1.14%	0.00%

Table 2: Pick-up from external conductor vs position

4.2 Overlapping ends

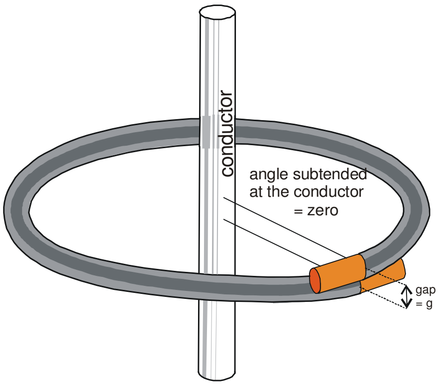


Figure 6: Coil with overlapping ends

A simpler alternative to using a compensation coil is to overlap the coil ends. This is shown in Figure 6. The ends of the winding are aligned above each other in the same direction as the conductor. There is a gap, g, between the ends but the direction of the gap is perpendicular to the plane of the coil. Provided the coil is mounted with its plane perpendicular to the conductor, the angle subtended at the conductor (θ_2 - θ_1 in Equation 1) is zero. The coil behaves as if there is no gap and the output is not sensitive to the position of the conductor.

If the conductor is external to the coil there will be no pick-up effects provided the plane of the coil remains perpendicular to the external conductor.

NOTE: In the figures illustrating this section the grey part of the coil represents the winding and the orange part is the unwound coil former.

4.2.1 Tilted Coil: The situation becomes more complicated when the plane of the coil is not perpendicular to the conductor. This is illustrated in Figure 7 which shows a coil tilted at an angle, λ. The 'effective gap' = g sin**λ** which is the component of the gap in the plane perpendicular to the conductor.

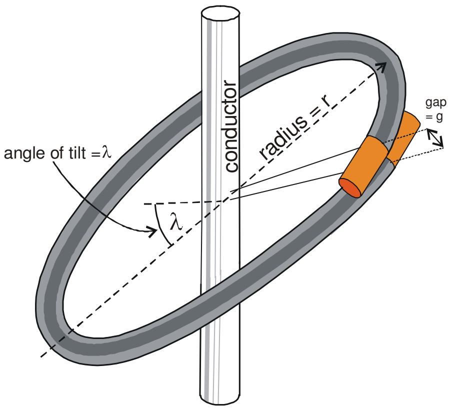


Figure 7: Tilted coil

For the case where the conductor is at the centre of the coil, the angle subtended at the conductor = g sinλ/r , where r is the radius of the coil.

The measurement error caused by tilting the coil is equal to the angle subtended by the gap compared with the angle subtended by the whole coil (= 2π).

$$\text{Measurement error} = \frac{1}{2\pi}\frac{g\sin\lambda}{r} = 100\frac{g\sin\lambda}{2\pi r}\% \qquad (4)$$

Using the same principle it is possible to calculate the measurement error when the conductor is not at the centre of a tilted coil.

Equation 4 shows that the measurement error is large when the ratio g/r is large. g is determined by the cross-sectional diameter, d, of the coil including any insulation. r depends on the length of the coil.

4.2.2 External Conductor: In Section 4.2 it was shown that, provided the plane of the coil is perpendicular to the conductor, the pick-up from an external conductor is zero. When the plane of the coil is parallel to the conductor the angle subtended by the gap is $\alpha = \frac{g}{D}$ where D is the distance of the conductor from the gap. The pick-up error is expressed as the pick-up voltage from an external conductor compared with the voltage output from the coil if the same conductor were threading it. The voltages are proportional to the angles subtended.

$$\text{Pick-up error} = \frac{\alpha}{2\pi} = 100\frac{g}{2\pi D}\% \qquad (5)$$

The size of the gap, g, is mainly determined by the cross-sectional diameter of the coil including any insulation. It follows that pick-up errors in coils with overlapping ends are lowest for coils with a small cross-section.

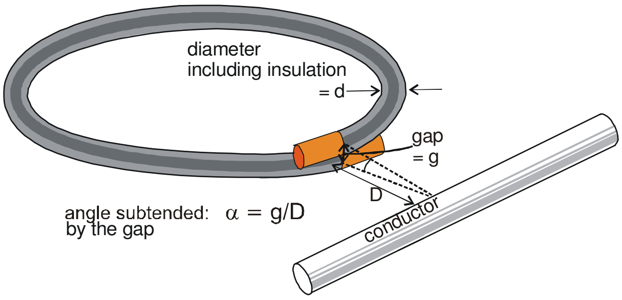


Figure 8: External conductor parallel to the plane of the coil

For the case where the plane of the coil is not parallel to the external conductor the pick-up error is proportional to the cosine of the angle between the plane of the coil and the conductor.

4.2.3 Multiple Wraps: One way to increase the output of a coil is to wrap it several times round the conductor. The output is increased in proportion to the number of wraps.

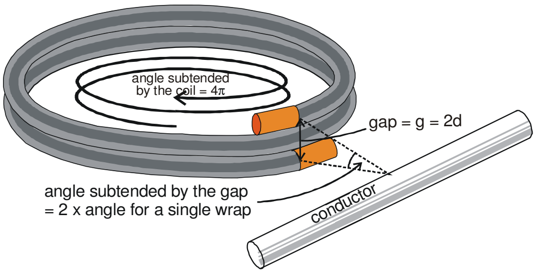


Figure 9: Coil wrapped twice

For two wraps the size of the gap is doubled but the number of times the coil is wrapped round the conductor is also doubled (Figure 9).

Consequently the level of pick-up error is the same as for a single-wrap coil. The same principle applies for coils which are wrapped three or more times.

For multiple wraps the gap between the ends of the winding can be reduced by bringing the ends of the coil closer together. Figure 10 shows an open helix where the gap is proportional to the number of wraps. As described earlier in this Section, the pick-up from an external conductor, expressed as a percentage of the coil output, is the same as for a single-wrap coil with overlapping ends (Fig 8). Figures 11a and 11b show two ways that the gap can be reduced by bringing the ends closer together. For these configurations the percentage pick-up from an external conductor is much lower than for a single-wrap coil.

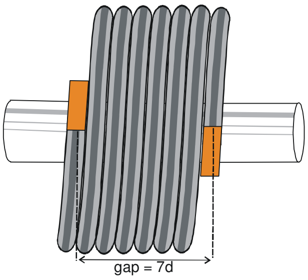
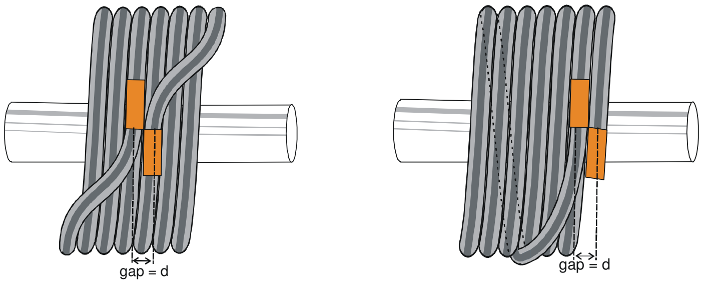


Figure 10: Open helix



Figures 11a, 11b: Bringing the ends closer together.

It is also possible to wrap the coil in two layers so that the end of the coil returns close to the start.

5. Other Defects

It has been assumed so far that the coil winding is completely uniform, consting of a uniform turns density on a former of constant cross-section, and that the only 'defect' is due to the gap in the winding where the ends come together. In a real coil the cross-section of the former may not be perfectly uniform and the winding turns may not be exactly perpendicular to the length of the coil.

5.1 Bulges and Thin Sections in the Coil Former

Bulges and thin sections will degrade the performance of the coil so that, even if the winding gap is correctly compensated, the coil will be subject to pick-up from an external conductor and to a change in output if the conductor is off-centre. Figure 12 shows a coil with a defective section of length L. This section has a strength, S_d, which is different from the strength, S, of the rest of the winding.

The defect can be considered as an additional winding with strength = $S_d - S$ superimposed on an otherwise perfect coil. In this way the effect of the defect can be described solely in terms of the additional winding.

For a conductor at a distance r from the defect, the angle subtended is: $\alpha_d = L/r$. Referring to equations 1 and 2 the contribution of the additional winding to the mutual inductance is given by: $M_d = L/r(S_d - S)$. The mutual inductance of the 'perfect' coil is $M = 2\pi S$. The effect of the defect is given by

$$\text{percentage error} = \frac{M_d}{M}\text{x}100 \;=\; \frac{L}{r}\,\frac{(S_d - S)}{2\pi S}\text{x}100. \qquad (5)$$

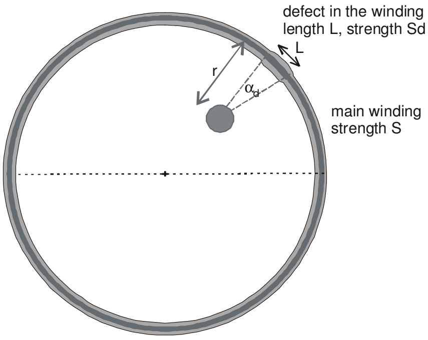


Figure 12: Coil with winding defect

The size of this effect is best illustrated by an example. The detailed geometry underlying this calculation is given in Part 1 Section 4.2.

Coil 160mm diameter, with a 20mm section where the cross-sectional diameter is increased by 10%.

a) Change in output when the conductor is moved 0.5 x coil radius (40mm) from the central position towards the defect = 0.8%.

b) Pick-up from an external conductor 160mm from the centre of the coil = 0.8%.

The same principle applies for a thin section in the winding in which case the 'additional winding', referred to above, has a negative strength. It can also apply if there is a region in the winding where the turns density is different from the main part of the coil.

5.2 Skewed Winding

This section considers the situation where the winding turns are not perpendicular to the length of the coil former. In this case a transverse magnetic field will induce a voltage. This is illustrated in Figure 13.

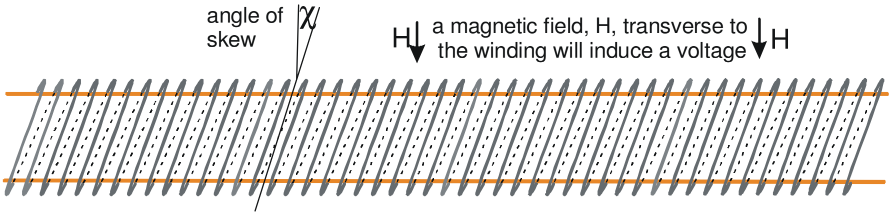


Figure 13: Skewed winding

If the cross-sectional area of the former is A the flux linked with each turn of the winding is given by

$$\text{flux per turn} = \mu_0 H A \tan(\chi) \qquad (6)$$

Where χ is the angle of skew. The effect of an external magnetic field depends on the direction the winding is skewed compared with the direction of the magnetic field. Equation 6 represents the worst case where the skewed winding presents the largest area to the magnetic field around the entire circumference. If the magnetic field were in a different direction relative to the direction of skew, the flux linked would be less, or zero, or negative.

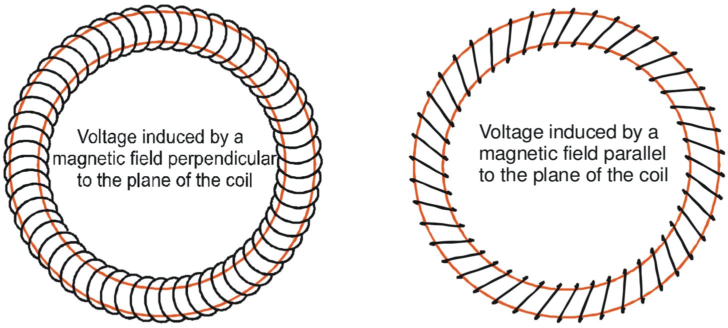


Figure 14: Angle of skew in relation to an external magnetic field

This is illustrated in Figure 14 which shows two extreme cases. However, in a real coil the direction of skew will change round the circumference of the coil so that some parts of the coil are sensitive to fields perpendicular to the plane of the coil and some are sensitive to parallel fields.

The best way to indicate the significance of a skewed winding is to consider a 'worst case'.

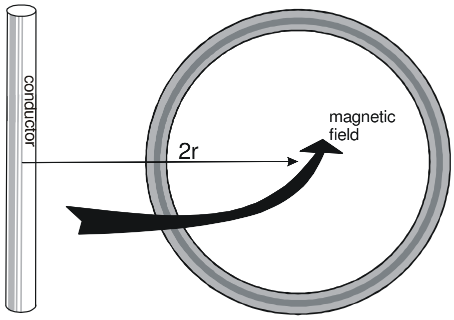


Figure 15a: Coil external to the conductor

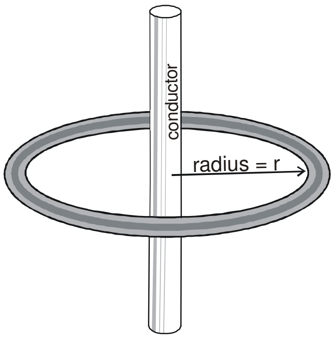


Figure 15b: The same coil encircling the conductor

The case considered is for a coil external to the conductor with the plane of the coil parallel to the conductor as shown in Figure 15a. It is assumed that the winding is skewed in a direction which links the maximum flux from the conductor. The flux linked is compared with the flux linked to the same coil encircling the conductor in the 'normal' measuring configuration (Figure 15b).

It is shown in Appendix 1, Equation A3, that the ratio between the pick-up voltage illustrated in Figure 15a and the normal output of the coil (Figure 15b) is given by:

$$\text{pick-up ratio (\%)} \quad = 57.7\tan(\chi)\%$$

From this equation is is seen that a skew angle (χ) of 1.0 degrees will give a pick-up ratio of about 1%. The main value of this result is not to give an accurate calculation of the pick-up voltage but to indicate how accurately the winding should be placed to avoid significant problems with skewed turns.

6. Conclusions

1) The theoretical approach described in Part 1 can be used to calculate the effect of various imperfections in the winding of a flexible Rogowski coil and to asses the effectiveness of different methods of wrapping the coil round a conductor.

2) In the construction of a Rogowski coil on a flexible former it is inevitable that there will be a gap in the winding where the ends of the coil are joined.

3) A gap in the winding causes the output of the coil to be sensitive to the position of the conductor threading the coil. It also makes the coil sensitive to interference or ‘pick-up’ from magnetic field sources external to the coil.

4) Two methods which have been used successfully to compensate for this gap are (a) using a compensation coil, and (b) overlapping the ends.

5) Use of a correctly-designed compensation coil can make the coil much less sensitive to conductor position and interference from external conductors. The compensation coil method is effective for all orientations of the coil with respect both to the conductor being measured and to an external magnetic field.

6) Use of overlapping ends can also make the coil much less sensitive to conductor position and external magnetic fields. However, to be most effective, the plane of the coil has to be perpendicular to the conductor being measured or to any external conductors.

7) The overlapping ends method is most effective with coils that have a small cross-section.

8) A coil can be wrapped several times tound the conductor to increase the output. If the coil is wrapped as an open helix the percentage errors due to pick-up and off-centre conductors are the same as for a single-wrap coil. These errors can be reduced by bringing the ends closer together.

9) Winding defects in coils include non-uniformities in the diameter of the coil former as well as missing turns and gaps. The effect of these can be estimated using the methods described in Part 1.

10) A 'skewed' winding can occur if the individual turns are not perpendicular to the direction of the former. This can give an error due to pick-up from an external conductor when the external conductor is parallel to the plane of the coil.

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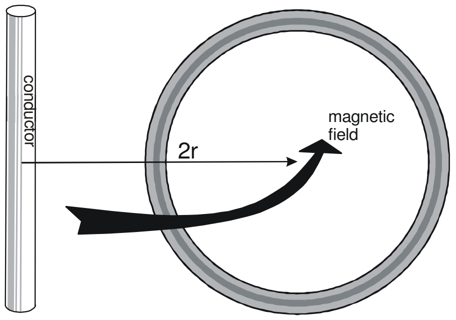
APPENDIX 1 Effect of a Skewed Winding



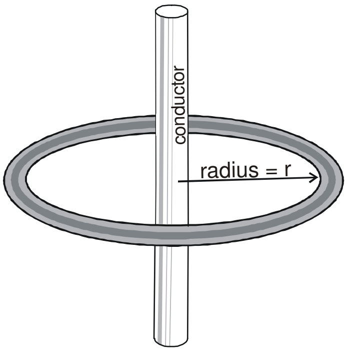


Figure A1: Coil external to the conductor

Figure A2: The same coil encircling the conductor

We consider a coil wound on a former that has a cross-sectional area A. Referring to Figure 13 we assume that the winding is skewed at an angle χ which gives the maximum flux linkage with the conductor. If the current in the conductor is I the approximate value of magnetic field perpendicular to the plane of the coil is given by $H = I/4\pi r$. Referring to equation 6:

$$\text{Flux linked per turn for an external conductor (pick-up flux)} = \frac{\mu_0 IA}{4\pi r}\tan(\chi) \qquad \text{(A1)}$$

Referring to figure A2 the magnetic field at the winding for the coil in normal measuring position is given by $H = I/2\pi r$.

$$\text{Flux linked per turn for normal measurements} = \frac{\mu_0 IA}{2\pi r} \qquad \text{(A2)}$$

Both coils have the same number of turns so the ratio of the pick-up flux from an external conductor compared with the normal output from the coil is found by dividing Equation A1 by Equation A2.

$$\text{pick-up ratio} = \frac{\tan(\chi)}{2} = 100\frac{\tan(\chi)}{2}\%$$

The derivation of equation 1 uses the approximation that the magnetic field at the winding equals the field at the centre of the coil. In fact the magnetic field will vary along the length of the winding depending on the distance of a particular section of the winding from the conductor. If the calculation is done 'properly' this introduces a factor of 1.155, giving:

$$\text{pick-up ratio (\%)} = 57.7\tan(\chi)\% \qquad \text{(A3)}$$